A method for numerical modeling of tsunami run-up on the coast of an arbitrary profile

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Abstract. In this paper a new method for numerical simulation of the long wave run-up process is proposed. Nonlinear shallow water equations are used to describe wave propagation up to the water-edge point. Then a special algorithm is used to estimate flow parameters and location of the moving water-edge point. It is based on energy and mass conservation laws. Several series of one-dimensional computations were carried out. A shore profile, which gives the maximum run-up height for the fixed initial wave parameters, has been found. Results of modeling tsunami run-up on the real shore in the Akita prefecture (Japan) are presented.

1. Introduction

One of the most important questions in prognostic tsunami modeling is estimation of tsunami run-up heights at different points along the coastline. Methods for numerical simulation of tsunami wave propagation in deep and shallow seas are well developed and are widely used by a great number of scientists. Some of them, in order to find possible submerged areas, use the simplifying assumptions about the ratio between the tsunami wave height near the shore and wave run-up height. But this ratio is heavily dependent on the shore profile above mean sea level.

2. Statement of the Problem

In this paper the method for numerical calculation of a tsunami wave run-up on a shore of an arbitrary profile will be described. Numerical modeling of this process was carried out on the basis of the one-dimensional nonlinear shallow-water model

$$\begin{aligned} u_t + uu_x + g\eta_x &= 0, \\ \eta_t + (u(\eta + H))_x &= 0. \end{aligned} \tag{1}$$

Here u is velocity, η is surface elevation, H is the value of depth, and g is gravity acceleration. The statement of the problem is as follows: there is a one-dimensional nearcostal area with an arbitrary bottom relief. From the left boundary of the computational area the tsunami wave is coming toward the shore. Above the mean sea level the coast is of an arbitrary profile (Fig. 1). In numerical computation, the bottom and coastal relief is given

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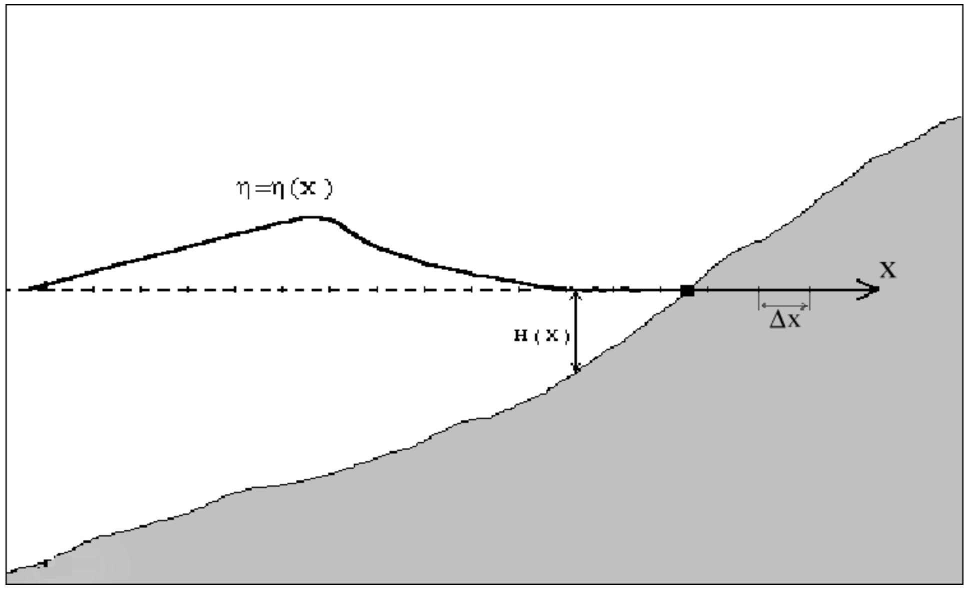

Figure 1: Statement of the one-dimensional problem of tsunami run-up on a shore.

as an array of elevation values in grid points. In the beginning the water in the regarded area is still, so the initial conditions can be written as

$$u(x,0) = \eta(x,0) = 0. \tag{2}$$

The incident tsunami wave is generated by motion of water and a free surface on the left boundary of the computational area

$$\begin{aligned} \eta(t) &= 1 + \sin(b \cdot t - \pi/2), \\ u(t) &= \eta(t) \cdot \sqrt{g/H}, \\ t &\in \left(0,\ \frac{2\pi}{b}\right). \end{aligned} \tag{3}$$

After the wave is formed completely (which happens when time value t becomes equal to $2\pi/b$), the "free" boundary conditions are used on this boundary

$$\frac{\partial \eta}{\partial x} = 0, \frac{\partial u}{\partial x} = 0. \tag{4}$$

3. Numerical Algorithm

The differential problem (1)–(4) is solved numerically with the help of the method of finite differences. All the variables are defined in the grid-points and the values of depth and coastal point elevation is given as the array $H_i (i = 1, \ldots, M)$. The explicit finite difference equations with central dif-

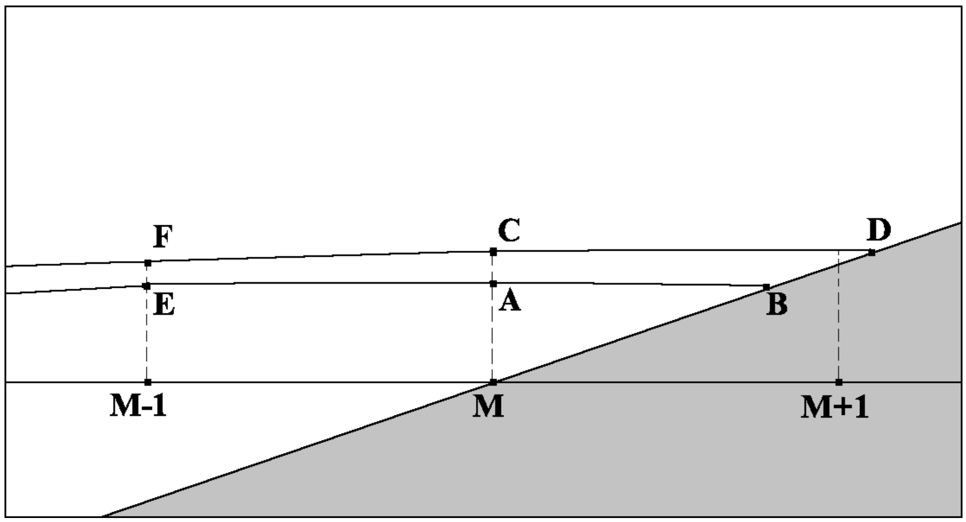

Figure 2: Definition of the water-edge point location.

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$$\frac{u_i^{n+1} - u_i^n}{\Delta t} + u_i^n \frac{\left(u_{i+1}^n - u_{i-1}^n\right)}{2\Delta x} + g \frac{\left(\eta_{i+1}^n - \eta_{i-1}^n\right)}{2\Delta x} = 0,$$

$$\frac{\eta_i^{n+1} - \eta_i^n}{\Delta t} + (H_i + \eta_i^n) \frac{u_{i+1}^{n+1} - u_{i-1}^{n+1}}{2\Delta x} + u_i^{n+1} \frac{H_{i+1} + \eta_{i+1}^n - H_{i-1} - \eta_{i-1}^n}{2\Delta x} = 0, \tag{5}$$

are used for computations in the inner points of the area. Values of the velocity $u_i^{n+1} (i = 1, \ldots, M)$ are preliminary defined from the first equation of (5) in the entire computational area. Then from the second equation (5) one can define the values of the surface elevation η_i^{n+1} on the higher time level in all inner grid points.

On the right boundary (the coast) where the depth value is equal to zero or is very close to this value ($\sim$0.01 m) the "free" boundary conditions (4) are used until the moment when the surface disturbance will reach this boundary grid point. From this moment the special algorithm for computation of the flow parameters is used.

We introduce the new computational point **B**, which shows the location of the water-edge point in every discrete time moment (time step). The position of this point is determined by calculating the volume of water, which comes through the cross-section **MA** (Fig. 2) during every time interval. We assume that the surface profile between points **A** and **B** represents a straight segment. According to this, the location of point **B** can be calculated from the total water volume that has flowed through the cross-section **MA** in the shore direction.

The distance between a projection of a point **A** and projection of a water-edge point **B** cannot exceed one computational spatial step. The change of a volume on each step is summarized and on a basis of a full volume (square of a triangle **MAB**) the new position of the water-edge point is determined.

The scheme of this algorithm is drawn in Fig. 2. Let points **E** and **A** in some time moment show the position of the free surface in computational grid points with numbers $M-1$ and M accordingly and point B-position of the water-edge point in this moment. Elevation of the free surface and water velocity in the grid-point number $M-1$ are calculated by difference scheme (2). In the grid-point number M (point **M** in Fig. 2) this can be done using the following procedure. At first we define new values of velocity and elevation $\left(u^{n+1}_{M-1/2} \text{ and } \eta^{n+1}_{M-1/2}\right)$ in a midpoint between grid-points number $M-1$ and M using difference equations

$$\begin{aligned}
&\frac{u^{n+1}_{M-1/2}-u^{n}_{M-1/2}}{\Delta t}+\frac{\left(u^{n}_{M-1}-u^{n}_{M}\right)}{2}\cdot\frac{u^{n}_{M}-u^{n}_{M-1}}{\Delta x}+g\frac{\eta^{n}_{M}+\eta^{n}_{M-1}}{\Delta x}=0,\\
&\frac{\eta^{n+1}_{M-1/2}-\eta^{n}_{M-1/2}}{\Delta t}+\frac{\left(H_M+\eta^{n}_{M}+H_{M-1}+\eta^{n}_{M-1}\right)}{2}\cdot\frac{u^{n+1}_{M}-u^{n+1}_{M-1}}{\Delta x}\\
&\qquad\qquad+u^{n+1}_{M-1/2}\frac{H_M+\eta^{n}_{M}+H_{M-1}+\eta^{n}_{M-1}}{\Delta x}=0.
\end{aligned}\tag{6}$$

And in the midpoint between points **A** and **B** $\left(\text{values } u^{n+1}_{M+1/2} \text{ and } \eta^{n+1}_{M+1/2}\right)$:

$$\begin{aligned}
&\frac{u^{n+1}_{M+1/2}-u^{n}_{M+1/2}}{\Delta t}+\frac{\left(u^{n}_{M}+u^{n}_{B}\right)}{2}\cdot\frac{u^{n}_{B}-u^{n}_{M}}{R}+g\frac{\eta^{n}_{B}+\eta^{n}_{M}}{R}=0,\\
&\frac{\eta^{n+1}_{M+1/2}-\eta^{n}_{M+1/2}}{\Delta t}+\frac{\left(H_M+\eta^{n}_{M}+H_{B}+\eta^{n}_{B}\right)}{2}\cdot\frac{u^{n}_{B}-u^{n}_{M}}{R}\\
&\qquad\qquad+u^{n+1}_{M+1/2}\frac{H_B+\eta^{n}_{B}+H_{M}-\eta^{n}_{M}}{R}=0.
\end{aligned}\tag{7}$$

Then with the help of a linear interpolation the values of velocity and elevation on the new time level can be defined in the grid-point with number M.

$$\begin{aligned}
u^{n+1}_{M}&=\frac{\left(\frac{\Delta x}{2}\cdot u^{n+1}_{M+1/2}+\frac{R}{2}\cdot u^{n+1}_{M-1/2}\right)}{(R+\Delta x)/2},\\
\eta^{n+1}_{M}&=\frac{\left(\frac{\Delta x}{2}\cdot \eta^{n+1}_{M+1/2}+\frac{R}{2}\cdot \eta^{n+1}_{M-1/2}\right)}{(R+\Delta x)/2}.
\end{aligned}\tag{8}$$

Here the value **R** represents distance along the horizontal axis from the right-most computational grid-point up to a current position of the water-edge point, and variables with index B are the values of flow parameters in the mobile point of the water edge. The water-edge velocity is determined using the energy conservation law. As noticed earlier, its location is determined according the mass conservation law. If the distance between projections of points **C** and the water-edge point becomes greater than a step of the spatial computational grid (point **D** in Fig. 2), then we introduce the new grid-point with number $M+1$ and the computational algorithm moves to this new grid-point.

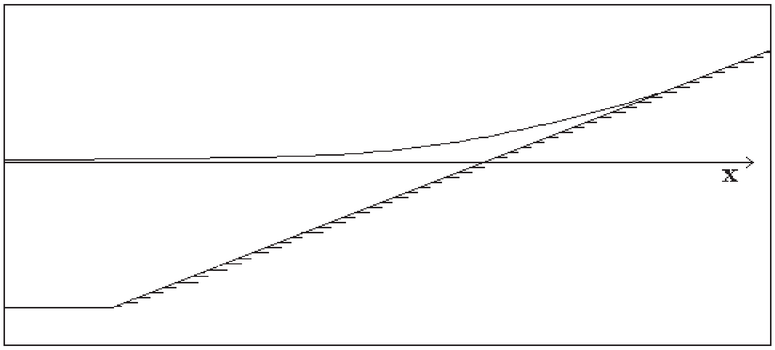

Figure 3: Water surface at the moment of highest run-up.

4. Results of Numerical Experiments

Using this computational method a number of computational experiments of a tsunami run-up on a shore of an arbitrary profile were carried out. The main goal of this computation was to test this algorithm using results of other methods (Pelinovsky, 1982; Marchuk, 1982; Pelinovsky, 1985) and investigate the shore profile influence on wave run-up height. The first series of test experiments was carried out for an inclined shore with the following parameters:

Spatial step of the grid:	$dx = 10$ m.
Initial number of computational grid-points:	$M = 400$.
Time step:	$dt = 0.2$ s.
Length of an incident wage (3)	$b = 0.1$.
Initial wave height	$\eta_0 = 4$ m.

The wave was generated on the distance of 4 km off the shore. The profile of the bottom was the following: from the water edge point the bottom has a constant inclination angle ($tg\alpha = 0.1$) and the depth increases linearly up to 100 m. Then the depth is not varying until the left boundary. In Table 1 the wave run-up heights at different beach slopes are shown.

Table 1: Wave run-up heights at different beach slopes

	Beach slope angle $tg(\alpha)$									
	0.1	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**	**0.8**	**0.9**	**1.0**
Run-up height	14.01	14.04	13.92	13.49	13.18	12.88	12.56	12.57	12.59	12.60

As it approaches the coast, the wave height increases up to 5.9 m. From Table 1 it is apparent that the increase of an angle of the shore slope causes a decrease of the wave run-up height, that corresponds to the results obtained earlier with the help of analytical and numerical methods (Pelinovsky, 1982; Marchuk, 1982; Pelinovsky, 1985). Figure 3 shows the wave profile at the moment of maximum run-up.

Now we shall consider tsunami wave run-up on a shore with a more complicated profile. The second series of numerical experiments is carried out in order to investigate the correltaion between wave run-up height and the profile of the shore submerging area. In this case, the profile of the submerging area of the shore (up to 25 m above mean sea level) is defined by the following expression:

$$H(r) = r \cdot tg(\alpha) - C \cdot \sin\left(\frac{\pi \cdot r}{r_0}\right), \ 0 \le r \le r_0. \tag{9}$$

Here the distance r is measured in the right direction from the initial water-edge point, r_0 is the width of a shore area with varied inclination (in this series of calculations it was equal to 250 m), and C is the curvature parameter. All remaining parameters are the same, as they were in the former case. As the result of computations with the positive values of parameter C, the run-up heights were obtained (Table 2). In this series of computations, the wave height near the shore was equal to 7.6 m.

Table 2: Wave run-up heights at different curvature parameters.

	Curvature parameter C							
	0.0	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**	**0.6**	**0.7**
Run-up height	23.067	23.086	23.132	23.73	23.86	23.54	23.68	23.63

The shore and the water surface profile when the curvature parameter C is equal to +4 are shown in Fig. 4. Results of computations with negative values of the curvature parameter C are shown in Table 3.

Table 3: Wave run-up heights at different curvature parameters.

	Curvature parameter C					
	–1.0	**–2.0**	**–3.0**	**–4.0**	**–5.0**	**–6.0**
Run-up height	22.58	22.82	22.40	22.37	21.88	20.86

The shore and the water surface profile for the curvature parameter value is equal to −6 and is shown in Fig. 5. From Tables 2 and 3, it is seen that the maximum run-up height of the wave with given initial parameters is observed when parameter C is equal to +4.

Numerical modeling of the long wave run-up on the real shore was carried out. The profile of the Japan coast (Fig. 6) was taken from the Survey Report of Tsunami of 26 May 1983 Along the Coasts of Akita Prefecture (1984). In this place (near Minehama village), run-up height was recorded as 14.08 m. To reach such a height, the incident tsunami wave with a period of 65 s must be about 8 m high near the coastline. The wave profile in this case is shown in Fig. 7.

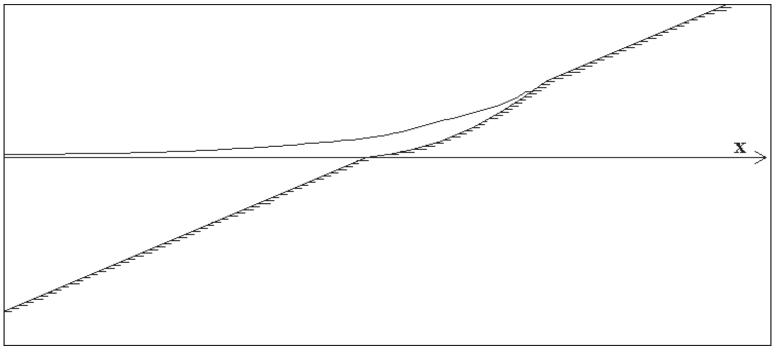

Figure 4: Wave run-up on a shore with the positive curvature parameter ($C = 4$).

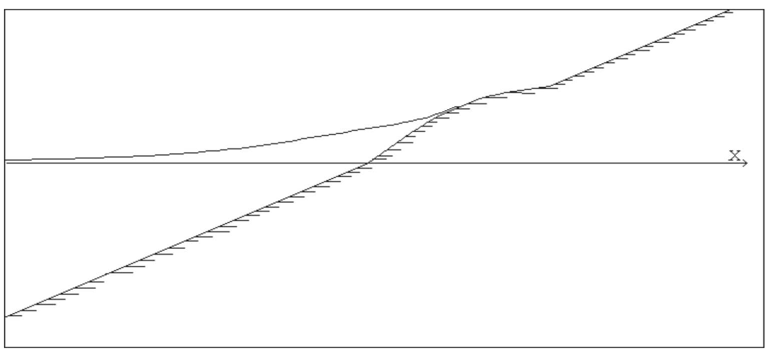

Figure 5: Wave run-up on a shore with the negative curvature parameter ($C = -6$).

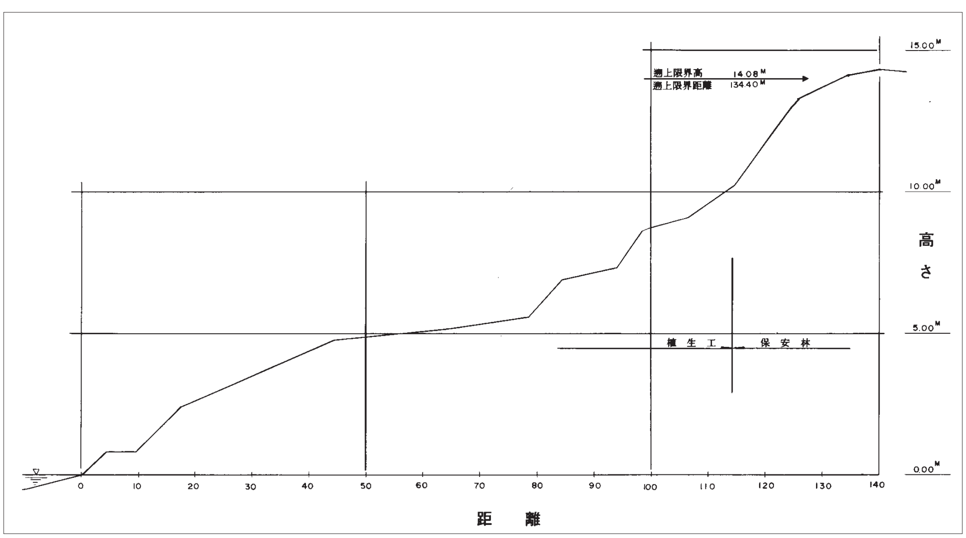

Figure 6: Shore profile near Minehama village in Akita prefecture (Japan).

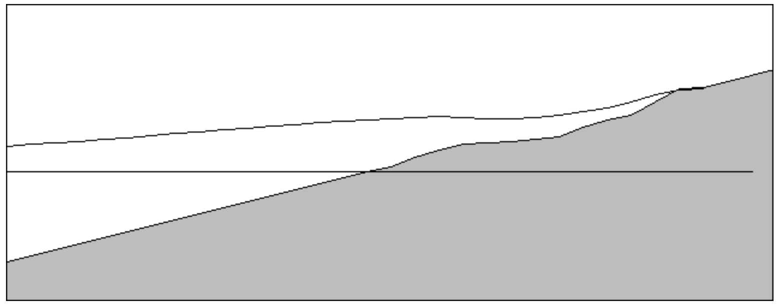

Figure 7: Modeling of Japan Sea Tsunami 26 May 1983 at the Akita coast. Tsunami wave profile at the moment of highest run-up.

5. Conclusions

A new algorithm of tsunami wave run-up computation on the shore of an arbitrary profile is developed. The results of the wave run-up on a sloping shore are in agreement with the results obtained earlier with the help of analytical and numerical methods. The computational experiments of a tsunami impact on a shore of an arbitrary profile have revealed a certain type of shore relief which gives the highest climb of the water on the shores of various profiles with an identical initial wave. The tsunami wave run-up on a shore simulation program describes precisely enough the behavior of a wave and can be used in simulation of real tsunami waves.

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6. References

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