A numerical technique for calculation of tsunami generation, propagation, and inundation of dry land

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Abstract. Tsunamis occur due to regional tectonic tilting, abrupt fault displacement, rockfalls into water, submarine landslide/volcanic eruptions, or meteorite impacts. The discrete element method is used to model the dynamic behavior of bodies of water due to the above causes. With this technique, a water body and its underlying topography are modeled by an assemblage of discs and quadrilateral elements, respectively. A computer program is developed to calculate the motion of each zone (disc/element) as a function of time. When the input disturbance begins, the zones start moving and exerting contact forces on the adjacent zones. Once the contact forces are calculated for a zone, Newton's second law is invoked to compute the zone accelerations, which are then numerically integrated to get velocities and integrated once again to get displacements. With this new set of displacements the calculation cycle is repeated. In this repetitive manner the assemblage of zones moves in space and marches forward with time. As time proceeds, the dynamic equilibrium of the assemblage of zones develops naturally. The analysis results (e.g., rockfalls into a reservoir, triggered by an earthquake) demonstrate the evolution of an impulsive generated tsunami from its generation and propagation to inundation of dry land.

1. Introduction

Tsunamis can be generated by several mechanisms. Most tsunamis are associated with submarine seismic disturbances. However, it is known that the explosion of an underwater volcano can cause one, or an island exploding, such as Krakatoa (Wiegel, 1964), or meteorite impacts (González, 1999). An undersea earthquake, characterized by motion along a thrust or normal fault, forces many thousands of square miles of ocean floor to lurch upward or downward in terms of many feet, which pushes up or drops down the overlaying column of water, respectively. Once large overlying bodies of water are displaced a disastrous tsunami is born. Tsunamis may also be generated indirectly by seismic vibrations, which may trigger other seafloor disturbances, such as rockfalls and submarine landslides, or an explosion of gas hydrates (González, 1999). Rockfalls or meteorite impacts disturb the surface of water, whereas submarine landslides, volcanic eruptions, or explosion of gas hydrates disturb deep water (González, 1999). A classic example of the magnitude of a wave that can be caused by a large body of rock sliding into a bay occurred in Lituya Bay, Alaska (Miller, 1960).

According to González (1999), regardless of their origin, tsunamis evolve through three overlapping but quite distinct physical processes: generation by any force that disturbs the water column, transoceanic propagation from deeper water or shallow coastal areas near the source, and finally inundation of dry land. Accurate simulations are important in predicting where future remote-source tsunamis will strike and in guiding disaster surveys and rescue

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efforts, which must concentrate their resources on regions believed to be hardest hit (González, 1999). Once a tsunami is generated by an earthquake or other causes, it propagates and transports destructive energy thousands of kilometers away from the source through the undulation of water until it reaches shorelines, where the last stage of a tsunami's evolution begins. Once the tsunami runs ashore as a breaking wave, it manifests itself as a wall of water or a tidelike flood and inundates the dry land. Vertical run-up can reach many feet. However, horizontal inundation, if unimpeded by coastal or other steep topography, can penetrate hundreds of feet inland.

The tsunami evolution from its inception to inundation of dry land is generally an exceedingly complex hydrodynamic problem. Historically, it was treated by analyzing only one selected feature of the problem in a rather idealized form. For example, the run-up problem according to Heitner and Housner (1971) falls into two classes: experimental model studies, and calculations based on various mathematical models. With respect to the mathematical models, researchers have used expansion solutions for certain types of waves, e.g., linear wave theory, nonlinear long waves, etc. Customarily, the small amplitude or linear wave theory is used offshore during the transmission phase and with nonlinear long wave theory used near the beach during inundation phase.

In the offshore area, the wave height is so small compared with both the wave length and the water depth that one can apply linear wave theory, which assumes that the height itself does not affect the wave's behavior (González, 1999). The linear theory describes purely oscillatory waves; i.e., the water particles move forward and backward with no mass transport as the waves pass by (Wiegel, 1964). The theory predicts that the deeper the water and the longer the wave, the faster the tsunami. This dependence of wave speed on water depth means that refraction by bumps and grooves on the sea floor can shift the wave's direction, especially as it travels into shallow water. In particular, the wave fronts tend to align parallel to the shoreline so that they wrap around a protruding headland before smashing into it with greatly focused incident energy (González, 1999). At the same time, each individual wave must also slow down because of decreasing water depth and bottom friction, so they begin to overtake one another, decreasing the distance between them. This process squeezes the same amount of energy into a smaller volume of water, creating higher waves and faster currents. As the tsunami waves get close to shorelines, the individual wave height is now so large that linear theory fails to describe the complicated interaction between the water and the shoreline (González, 1999). As such, the water particles move back and forth, but each particle moves farther forward than it does backward, and the particle is translated a small net amount forward with the passage of each wave (Wiegel, 1964). As a result, there is a net mass transport, which linear theory cannot account for. Therefore, as the wave gets into shoaling water, it progressively changes its characteristics as it propagates from deep water-linear theory to transitional and shallow water-nonlinear theory. As such, the nonlinear long wave theory is used near the beach. However, the nonlinear wave theory tends to predict breaking sooner than it is observed experimentally; in fact, it causes every waveform to break eventually (Heitner and Housner, 1971).

Heitner and Housner (1971) studied the run-up problem by accounting for the influence of bottom friction and of breaking waves so that the calculated energy and momentum transported to the beach by the wave are sufficiently accurate to calculate run-up. Since the physical domain of the run-up problem changes with time, this causes complications in the numerical calculations in a Eulerian coordinates system, for mesh points must be added and subtracted during the solution. Therefore, Lagrangian coordinates were used. In addition, the water depth was not included as an independent mathematical variable in the analysis as this so complicates the problem as to make computation impractical. Using one-dimensional space and time, Heitner and Housner (1971) made a simplification which allows the equations describing the flow to be integrated vertically, yet preserves the physics required to model the run-up process. By constraining the horizontal fluid velocity to be constant over the depth, they included the kinetic energy of the vertical component of velocity, which was reasonable for long waves approaching a beach. For steeper waves, such as solitary waves, the true distribution of horizontal velocity deviates more from uniformity and the constrained flow was a more approximate description.

To avoid the above assumptions and simplifications and to facilitate the modeling of tsunami evolution from its inception to inundation of dry land, the discrete element method was used. To further facilitate the approach, the discrete element methodology was implemented using a two-dimensional computer program to carry out the analysis. The program is used to model, for example, rockfalls into a reservoir to simulate the entire evolution of an impulsively generated tsunami from its inception to inundation of dry land.

2. Methodology

The discrete element method was used to model the entire tsunami evolution from generation and propagation to inundation of dry land. In this method, bodies of water, rockfalls, a submarine landslide, and a meteor are modeled by an assemblage of discs and the surface of the basal topography by an assemblage of quadrilateral elements. The discrete element method is capable of describing static and dynamic behavior of assemblies of zones (discs or quadrilateral elements). The method allows large displacements and rotation of zones, including complete detachment of the zones. This feature is very important when the tsunami waves run ashore with wave heights so large that they break. It is the key feature of the method, which makes it possible to follow non-linear interactions of a large number of zones without the need for an iterative procedure. The Tsunami2D computer program was developed to carry out the analysis using the discrete element method. The method was based on the use of an explicit numerical scheme in which the interaction of the zones is monitored contact by contact and the motion of the zones modeled zone by zone. In the discrete element method, the interaction of the zones is viewed as a transient problem with states of equilibrium developing whenever the internal forces balance. As such, the contact forces and displacements of an assembly of zones are found through a series of calculations tracing the movements of the individual zones. These movements are the result of the propagation of disturbances through the medium originating either at basal topography due to a seismic event or other mechanisms. The numerical scheme was based on the use of an explicit Lagrangian formulation of momentum equations, representing Newton's second law of motion. The formulation inherently takes into account the mass conservation law and allows zones with fixed masses to translate and rotate in space. The equations of motion are cast into a discrete algebraic form, which is solved at the center of gravity of each zone (disc/element). The key mathematical basis and the numerical implementation of the program, which are discussed elsewhere by Cundall and Strack (1979), are briefly presented in the following sections.

2.1 Numerical solution process

In the discrete element method, the body of water, rockfalls, submarine landslide, and a meteor are divided into an assemblage of discs and the basal topography by an assemblage of quadrilateral elements, as shown in Fig. 1. The mass of a zone (disc/element) is allocated to a grid point located at its center of gravity. Each grid point represents three degrees of freedom, one horizontally, one vertically, and one rotationally. The size of a disc or a quadrilateral element determines the ability to resolve fine details of displacements. In general, the smaller the size, the more accurate the results. A disc/element can be visualized as a rigid element with a coating of springs and dashpots in normal and shear (tangential) directions at its periphery, as shown in Fig. 1. Spring and dashpot elements are used to provide interaction between zones in contact and dissipation of impact energy, respectively. For simplicity and clarity, only normal springs are shown in Fig. 1. The deformations of the individual discs/elements are assumed small enough in comparison with the deformation of an assemblage of discs/elements as a whole. The latter deformation is due primarily to the movements of the discs/elements as rigid bodies. In this method the disc/element zones are allowed to overlap one another at contact points. This overlapping behavior takes the place of the deformation of the individual discs/elements. The magnitude of the overlap is related directly to the contact force in the way explained below. It should be noted, however, that these overlaps are small in relation to disc/element sizes.

The spring and dashpot elements need a force-displacement law, which determines the magnitude of the contact forces by the overlap between neighboring zones. Once two zones are in contact, the spring and dashpot elements at the location of contact are activated. In the normal direction, the compressions of the contact spring and dashpot elements are determined by the relative displacements (overlap between the contacting zones) and the relative velocities between these two zones, respectively. In turn, the amount of overlap is directly controlled by the stiffness of the springs. Similarly, in the tangential direction, the relative sliding of two contacting zones determines

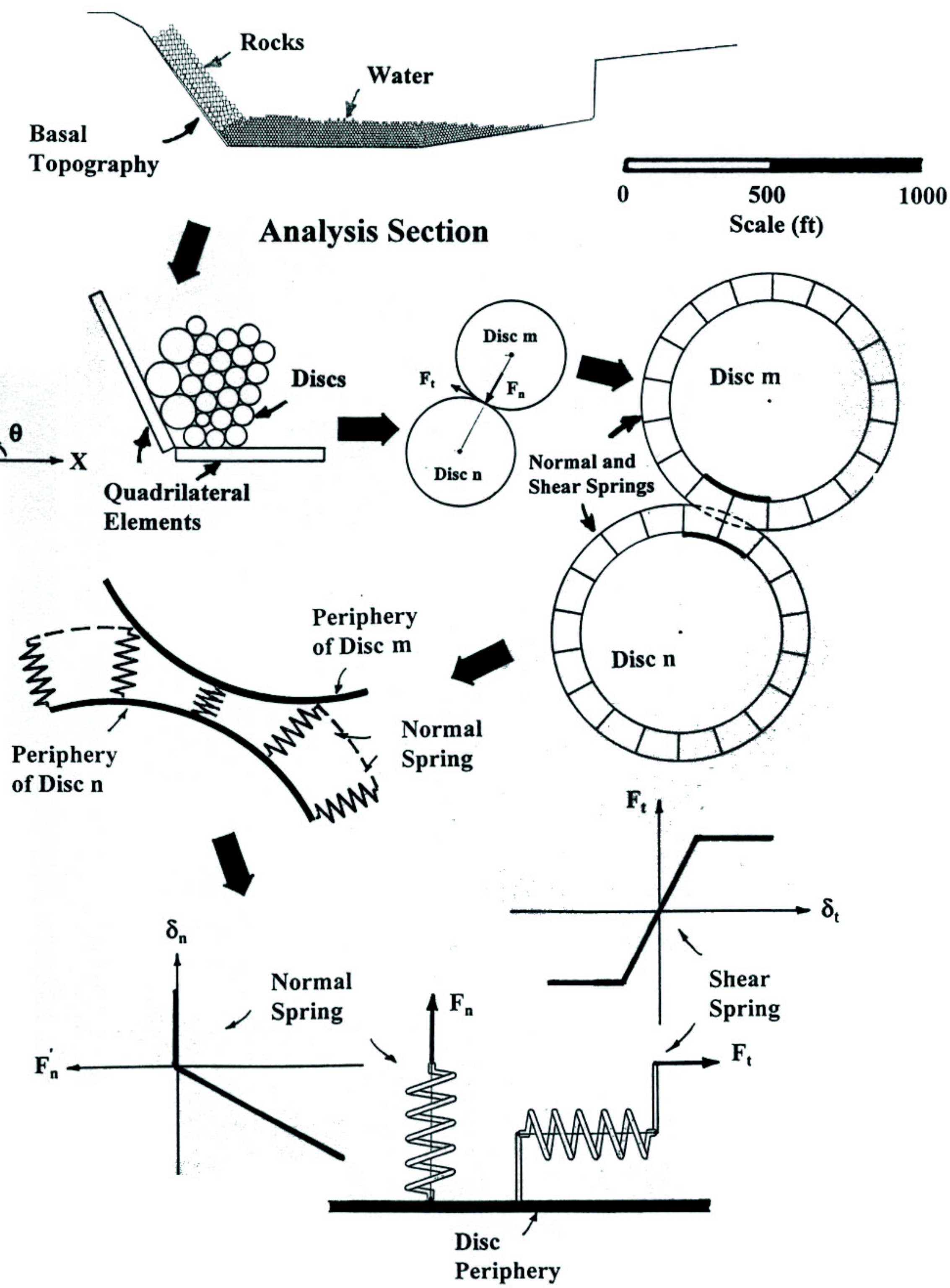

Figure 1: Schematic diagram for general methodology of discrete element method.

the force of the shear spring whereas the relative sliding velocity determines the force of shear dashpots. The calculations performed in the discrete element method alternate between the application of Newton's second law to the discs/elements and a force-displacement law at the contacts. Newton's second law gives the motion of a zone resulting from the acting forces. The force-displacement law is used to find contact forces from displacements. Once the contact forces built up on each zone are calculated, Newton's second law is invoked to compute the zone accelerations. The accelerations are then numerically integrated to get velocities and integrated once again to get displacements. With this new set of displacements the calculation cycle is repeated. In this repetitive manner the assemblage of discs/elements moves in space and the analysis marches forward with time. As time proceeds, dynamic equilibrium of the assemblage of discs/elements is developed naturally, satisfying both force equilibrium and displacement compatibility. Tsunami2D analysis always consists of two parts: (1) Turn-on gravity analysis to compute initial contact forces between zones due to gravity and (2) dynamic analysis to compute dynamic responses and deformations due to the effects of simultaneous gravity and either seismic disturbances or other causes.

2.1.1 Time step

The discrete element method is based on the idea that the time step chosen may be so small that during a single time step physical disturbances cannot propagate from any zone farther than its immediate neighbor. Then, at all times the resultant forces on any zone are determined exclusively by its interaction with the zones with which it is in contact. Suppose the shortest of such time steps is designated as the critical time step, and suppose the body of water and its basal topography, as shown in Fig. 1, are modeled as an assemblage of zones. Each zone is connected to its neighbors by springs at contact points. In such a system, the critical time step is controlled by the fundamental period of a zone connected to another zone by spring elements. To achieve a stable solution, the time step used in the explicit method must be smaller than the critical time step, which is proportional to the square root of the mass of the zone divided by the stiffness of the springs connecting the zone to its neighbors.

2.1.2 Initial boundary conditions

The computation processes are subject to appropriate initial and boundary conditions. The initial conditions normally consist of a specified initial geometry of zones having zero displacements, velocities, and accelerations at the beginning. The boundary conditions include zero acceleration, fixed acceleration, and acceleration or displacement history at the zones. In addition, the entire basal topography can be rotated appropriately to reflect a regional tectonic tilting, or some of its segments can be suddenly moved to reflect an abrupt fault displacement.

2.1.3 Material properties

Each zone has a material type, which is characterized by attributes that are relatively easily obtainable: geometry, total unit weight, drained and undrained shear strength, small strain bulk modulus, and shear modulus, if any. The shear strength for each soil or rock material type may be specified by means of a curved Mohr-Coulomb envelope. The viscosity of water may be specified by means of a cohesion value. Similarly, the friction between water and soil/rock materials can be specified either by some cohesion or friction value.

2.1.4 Force-displacement relationships

Force-displacement relationships are needed by spring elements at the periphery of two zones in contact. The program makes it possible to specify any force-displacement relationship. However, for this study, a linear relationship, or a spring constant, is used as shown Fig. 1. The value to be assigned to the spring constant is subject to two counteracting considerations: (1) keeping the spring constant relatively low to prevent the time step from becoming too small (resulting in an analysis that requires too much computational time), and (2) keeping the spring constant relatively high to reflect the expected compression wave velocity in the body of water or shear and compression wave velocities in the mass of soil/rock materials. For practical purposes, however, the analysis results are not sensitive to values of spring constants within a reasonable range. The force-deformation relationships are also subject to the following conditions, as shown in Fig. 1: (1) forces in shear springs are less than or equal to available shear strengths limited by the Coulomb friction law, and (2) forces in normal springs at contacts are less than or equal to zero. It should be noted that in the force-displacement relationships, normal forces in tension and compression are considered positive and negative, respectively.

2.1.5 Damping

Certain irreversible processes that convert kinetic energy to heat should take place between zones. This effect was approximately reflected in the analyses by allowing some damping in the system. The program includes two forms of viscous damping: local damping and global damping. Local damping operates on the relative velocities between two zones in contact. Global damping operates on the absolute velocities of the zones. Local dampings are represented by dashpot elements oriented in the shear and normal directions between two zones in contact, and global dampings are modeled by dashpot elements oriented in the horizontal and vertical directions connecting a zone to the inertial reference (Beikae, 1996).

3. Analysis Results

A two-dimensional reservoir with an unstable rock slope was considered for this study. As the top drawing in Fig. 2 shows, the length and the maximum

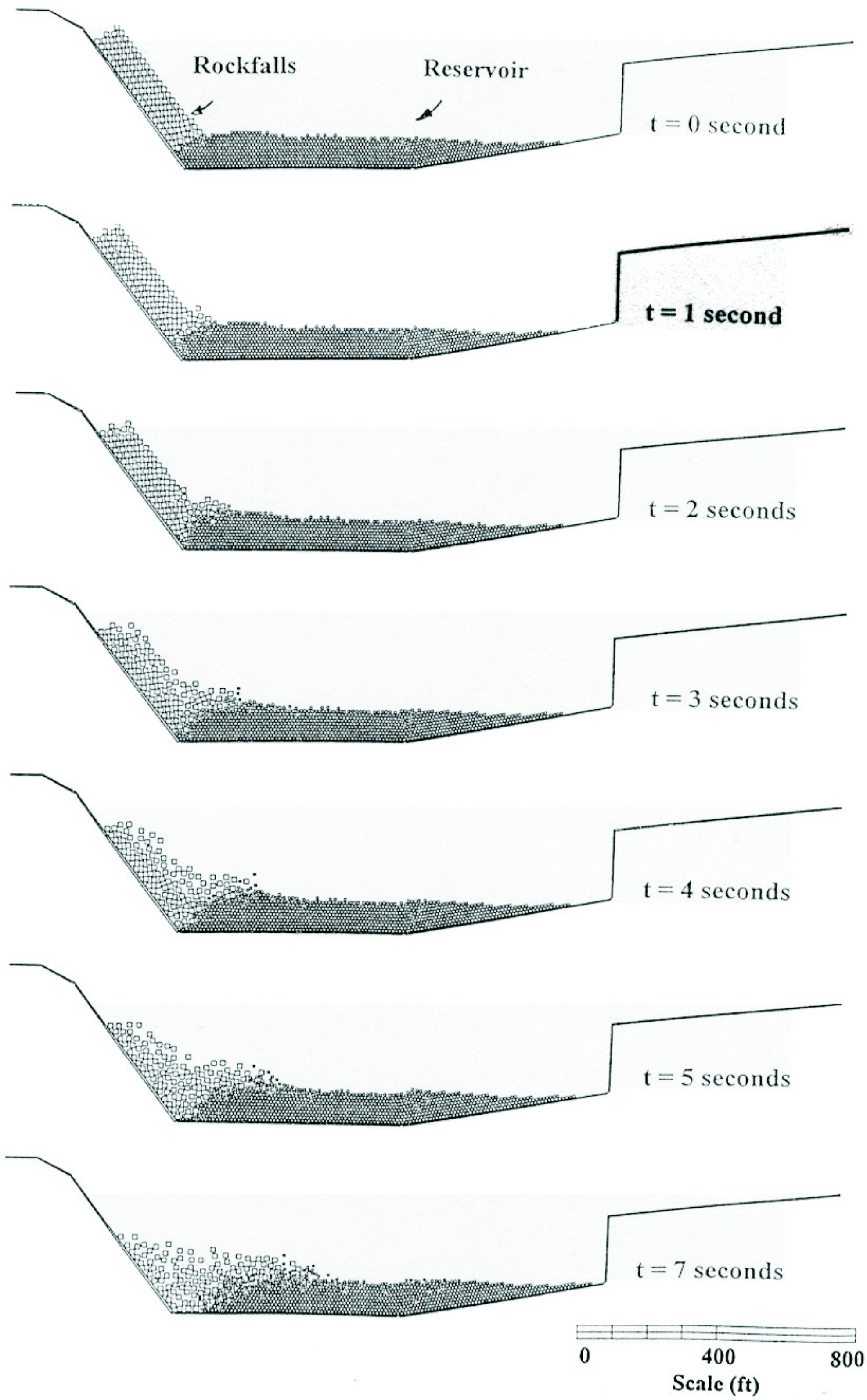

Figure 2: Variation of wave run-up profile with time due to rockfalls into reservoir.

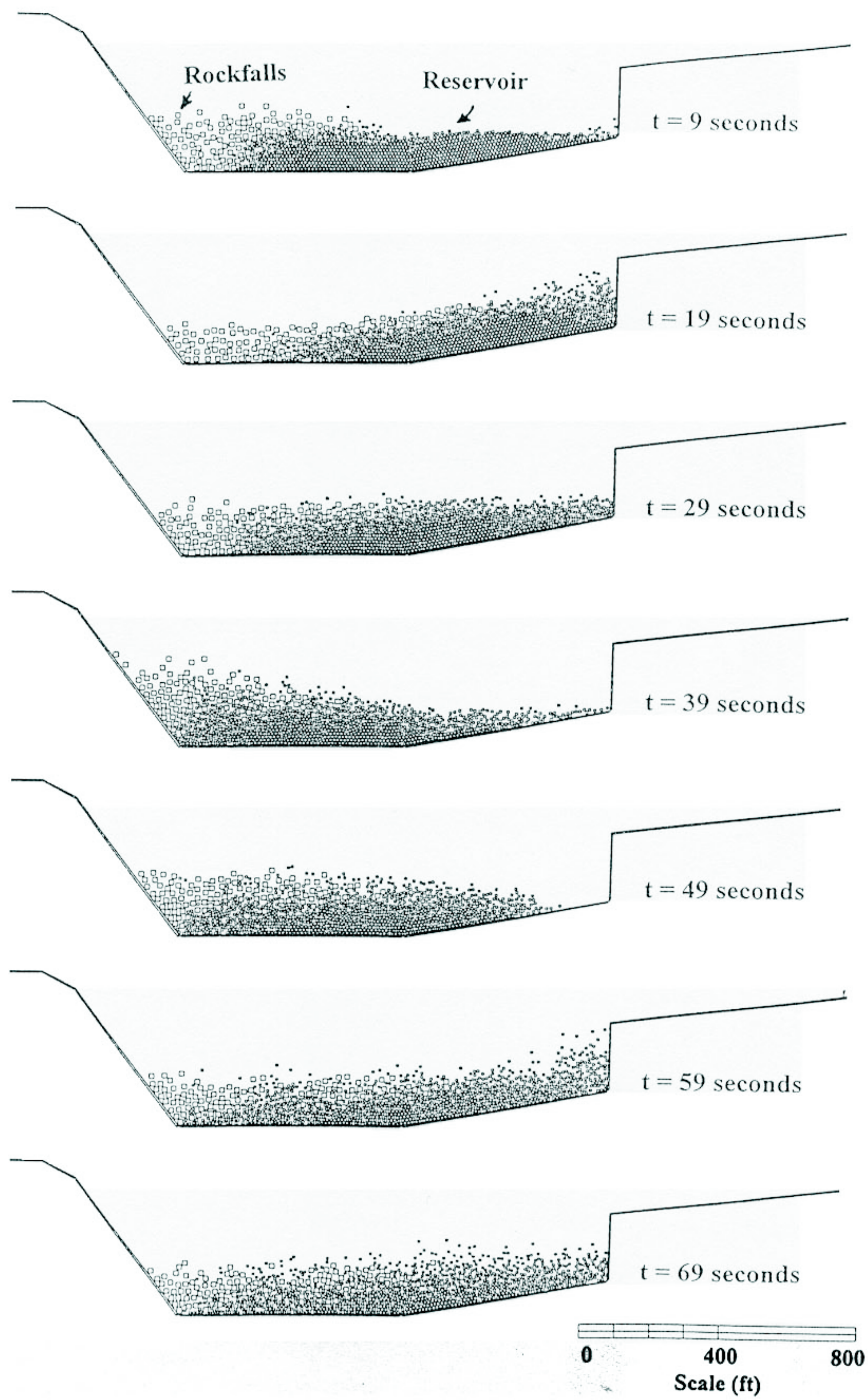

Figure 3: Variation of wave run-up profile with time due to rockfalls into reservoir.

depth of the reservoir are about 1100 feet and 100 feet, respectively. The horizontal distance between the rock slope on the left and the coastal cliff on the right is about 1250 feet. The horizontal length of a dry sloping shoreline next to the cliff is about 150 feet. The length of the unstable rock slope is about 400 feet and its thickness is about 100 feet. As shown in the top drawing of Figs. 1 and 2, the bodies of the water and the rock slope were modeled by 900 5-foot radius discs and 100 10-foot radius discs, respectively. The basal topography of the reservoir is also represented by seven thin quadrilateral elements. The total unit weight of water and the rock were considered 62.4 and 150 pcf, respectively, and the acceleration of gravity was 32.2 ft/s^2. The viscosity of water, the bottom friction, global damping, and local damping were all assumed zero in the analysis. Tsunami2D analysis was carried out in the analysis section discussed above. In the turn-on gravity analysis, the rock pieces located on a steep slope were assumed stable and fixed in their places. The top frame in Fig. 2 shows the analysis result indicating a stable and calm reservoir at t = 0. However, in the dynamic analysis, suddenly the rock pieces were set free with no internal cohesion and friction angle. As a result, the pieces of rock started falling into the reservoir under the force of gravity. Plunging into the water, rock pieces lost some kinetic energy due to impact and immediately got separated. In the meantime, water depth next to the falling rocks increased and an impulsive generated tsunami was born. As time proceeded, the tsunami wave started propagating to the right until it reached the shoreline. Figure 2 shows seven sequential frames of the tsunami generation and propagation across the reservoir for a total duration of 7 seconds. Having reached the shoreline, the tsunami started inundating the dry land and after about 150 ft of horizontal inundation, it encountered a 200-ft-high coastal cliff. It smashed into the cliff with greatly focused incident energy, as shown in a frame corresponding to t = 19 s. As shown in Fig. 3, the tsunami wave then reflected back to the left shoreline and after 60 seconds of back and forth oscillation the analysis was interrupted.

4. Conclusion

The Tsunami2D analysis is simple and realistic. Its results can be used to compliment experimental investigations. The advantage of this model lies in its flexibility, in that a variety of input disturbances, the influence of different bottom friction and bottom topography as well as irregular shorelines can be examined. The input disturbances are due to seismic events or other mechanisms. Although the program is written for two-dimensional space, with some modification and addition it can easily be extended to a three-dimensional one. The program can also provide the tsunami-induced dynamic forces on gravity structures, such as breakwaters and retaining walls, for design purposes.

5. References

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