Complex analysis of ocean tsunami observation data for solution of the inverse problem

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Abstract. A special system for mareogram processing is proposed. Such a system is based on two different approaches, namely, neural network technique, and inverse problems. By using two alternative methods, it is possible to achieve better accuracy in determining space parameters of a tsunami source. The above mentioned approaches are described in the paper. Model numerical tests, processed over the realistic depth profile, are then demonstrated.

1. Introduction

In this paper the inverse tsunami problem is considered, i.e., evaluation of parameters of a tsunami source based on data of observations of waves in the open ocean or on the shore. It is known that the solution of this inverse problem requires that during the analysis of mareograms the trend of the trace of wave propagation and the trend of the source of perturbation be extracted from tsunami records.

For construction of the trace function and the source function based on tsunami registration data, various methods can be used, such as analytic solution of the direct and inverse problems, numerical methods of modeling of excitation, and propagation of tsunami waves.

In this work a number of new approaches are proposed. They are based on complex optimization of the observation system, determination of the space distribution of tsunami source through the corresponding inverse problem, and nonlinear multiparameter regression analysis (neural-network technology) of tsunami wave records during which the trace function and the sought source function are reconstructed.

The informational-computational technology being created here is universal for the purposes of analysis of tsunami registration data. In particular, the optimization methods here applied allow one to solve the following particular tasks in the tsunami related problems:

- to compress measured data for effective transmission and storage of

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useful information without losses in the dynamic range during data recovery;

- to extract the informative signal from the noise and to estimate the characteristics of different phases of tsunami waves in on-line mode during their continuous registration;
- to estimate the threshold values of the amplitude and period of the tsunami wave in the problem of real-time prediction of tsunami danger;
- to find the optimal position of wave registration sensors in systems of early tsunami detection;
- to construct regression relationships between various parameters of the wave and characteristics of the site of underwater earthquake and source of tsunami.

2. Nonlinear Multiparameter Regression of Data

2.1 Neural-network approach of data analysis

Application of traditional mathematical methods in the investigation of tsunamis does not always give satisfactory results because the form of the sought relationships is *a priori* uncertain and nonlinear methods of analysis of observation data are relatively weakly developed.

Neural-network algorithms allow one to carry on the search for relationships in large arrays of data with arbitrary statistical distribution of variates. If relationships are revealed, then the neural-network model that was used for analysis can be applied for prediction and control of the process under study (Gorban, 2000).

In respect to information processing and analysis, neural networks differ from traditional approaches in the following points:

- availability of a flexible parallel-serial method of data processing in which the proportion of parallel and serial stages depends on a concrete problem;
- calibration of the neural model during solution of the problem instead of its programming;
- the stage of projection of a detailed calculation scheme is absent because that scheme consists of uniform relatively simple elements; moreover, the structure of connections between them is established during the calibration, which in the case of purely programmed realization corresponds to the application of one and the same simple computational module in programs oriented to the solution of different problems;
- the function performed by the neural network is specified by a special array of parameters that are formed during the calibration of the neural network;

- the block or program of learning and the system of modification of connections are an integral part of the neural-network program;
- high resistance to failures of elements or damage of the parameter array on a storage medium.

2.2 Description of the computational algorithm

Let us consider the computational technology of the construction of models of data with the aid of neural networks. The universality of such an approach is established by the known theorem: a collection of one nonlinear and many linear transformations is sufficient to approximate any single-valued function with an arbitrary given accuracy.

As was already mentioned, the neural-network approach—search for partial derivatives with the aid of the Lagrange multiplier method—allows one to construct such approximations rather effectively.

The setting of the problem of multiparameter nonlinear regression that is used in the present neural-network algorithm assumes that the relationship between known input variables and output variables being predicted is sought.

The quality of approximation is evaluated by the estimation function of the form

$$H = \sum_t h_t(x, A_t), \tag{1}$$

where h_t is the value of the estimate for the problem t, x is a set of parameters of regression, and A_t are input variables for the problem t.

The estimate for the problem t is most often defined by the square of the distance between measured output variables $\tilde{\alpha}_t$ and predicted output variables $\alpha_t(x, A_t)$

$$h_t = (\alpha_t(x, A_t) - \tilde{\alpha}_t)\varepsilon(\alpha_t(x, A_t) - \tilde{\alpha}_t). \tag{2}$$

In the above formula, the matrix ε assigns the coefficients of a nonnegative defined quadratic form; if this matrix is diagonal, then these coefficients are inversely proportional to tolerable root-mean-square deviations of predicted output variables from the measured ones.

The coefficients of ε may be called accuracy coefficients. If the inputs are not known exactly, then a natural generalization is to pass over to the estimation function

$$H' = H + \sum_t (A_t - \tilde{A}_t)\mu(A_t - \tilde{A}_t), \tag{3}$$

where $\tilde{A}_t$ are the measured input parameters which do not coincide with the best supposed parameters A_t obtained in the course of optimization of the estimation function H' with respect to these parameters, and μ are the coefficients of accuracy of input variables. The limit case when zero accuracy corresponds to some or all inputs deserves special consideration.

For convergence of the calibrating procedure that realizes minimization of the estimate, it is desirable that the set of parameters being changed during learning be necessarily compact. For this problem, one can choose variants of requirements of *a priori* compactness that admit of meaningful interpretation in terms of smoothness of the dependence of α_t on A_t. Namely, two variants of conditions may be used:

$$\begin{gathered} \left\langle (\nabla a_t)^2 \right\rangle \leq const; \\ \left\langle (\nabla a_t)^2 \right\rangle / \left\langle {a_t}^2 \right\rangle \leq const, \end{gathered} \tag{4}$$

that are imposed either on the average (over the space of inputs) values of squares of derivatives of the input variables with respect to the output ones (the first variant) or on the ratios of these average values to the average squares of the input variables (the second relation).

In this work, we used the neural-network approximation of the input-to-output dependence of the following form:

$$a_t^i = b^i + c^i \sum_q \sin\left(\varphi^{iq} + \sum_j k^{qj} \times A_t^j\right). \tag{5}$$

If the above approximation is chosen, then the first variant of the compactness condition corresponds to the requirement

$$\left(\sum_i (c^i)^2\right) \times \sum_{q,j} (k^{qj})^2 \leq const, \tag{6}$$

while the second variant corresponds to the requirement

$$\sum_{q,j} \left(k^{qj}\right)^2 \leq const. \tag{7}$$

In choosing a proper variant of neural-network approximation one often proceeds from the requirement of asymptotic universality of the neural network, i.e., the possibility to approximate arbitrary sufficiently smooth functions of input variables with arbitrary accuracy by increasing the number of network adjustment parameters.

One can show that when the number of adjustment parameters increases the above representation of the network transforms into a variant of the Fourier integral representation, whose approximational capabilities are well studied.

With the modified estimation functional, either a variant on the basis of the conjugate gradient method, when optimization is being done on the direct product of the space of parameters of the model and the space of input variables over all problems, or a variant in which modification of the neural-network model and modification of the input variables are carried on in turn were realized.

The reason for the application of the second variant is the possibility of carrying on adaptation on each problem in turn, which saves computational resources. According to the results of tests, in a number of cases one succeeds in satisfactorily reconstructing the missing information about inputs. At the same time, if the accuracy of input data is low, the operation of the algorithm is unstable, and additional regularization on the set of input variables is necessary.

2.3 Description of the software complex "MODELS"

The software complex "MODELS" that has been developed is intended for on-line synthesis of diverse information on tsunami by empirical and table data and also by results of calculations on the basis of analytic and numerical models with an adjustable level of smoothing of data.

It is assumed that synthesized analytical models reproduce approximately the cause-and-effect connections that are typical of the original object, to the extent in which these connections manifested themselves in the collection of empirical data or in numerical computations.

Having a number of neural-network models of the phenomenon at one's disposal, one can, instead of experiments with the original object, resort to numerical experiments for solution of applied problems in the problem of a tsunami with a corresponding constructed nonlinear multiparameter regression model.

Thus, from the mathematical viewpoint, the software complex performs nonlinear multidimensional regression with adjustable smoothness. As interpolation, one of the variants of multidimensional representations in the form of Fourier's integrals with integrals replaced by finite sums is used. The biarray of the neural-network model stores the parameters of the "optimal" finite Fourier-transform; the dimension of the model corresponds to the number of harmonics.

In optimization, the method of fast computation of multidimensional gradients, or the Lagrange multiplier method, which in the frames of the neural-network ideology is known as *back propagation*, and also the conjugate gradient method are used. Similar to all fast, predisposed to parallelism methods of nonlinear regression, the employed method can be called, by the adopted tradition, a neural-network one.

It should be noted that in the frames of neural-network ideology, the problem of multiparameter statistics being ill-conditioned is solved externally: poor conditionality may quite manifest itself in instability of parameters. But this instability is regularized in such a way that predictions and approximations in good cases, when according to the problem's sense their instability is not to occur, really turn out stable.

At present the regression analysis of mareograms for a number of historical tsunamis in the Pacific Ocean is being done with the aim of timely prediction of tsunami danger for certain parts of the Pacific shore of Russia (Figs. 1 and 2).

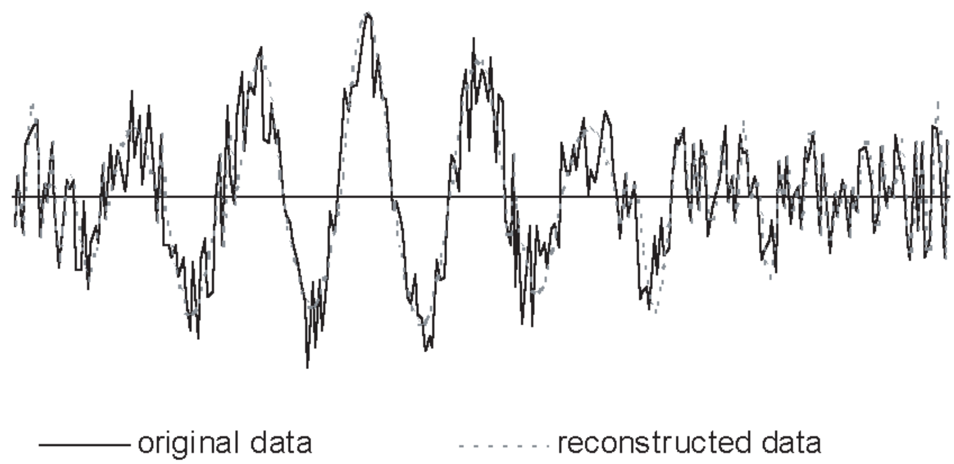

Figure 1: An example of compactization of a mareogram with a record of tsunami.

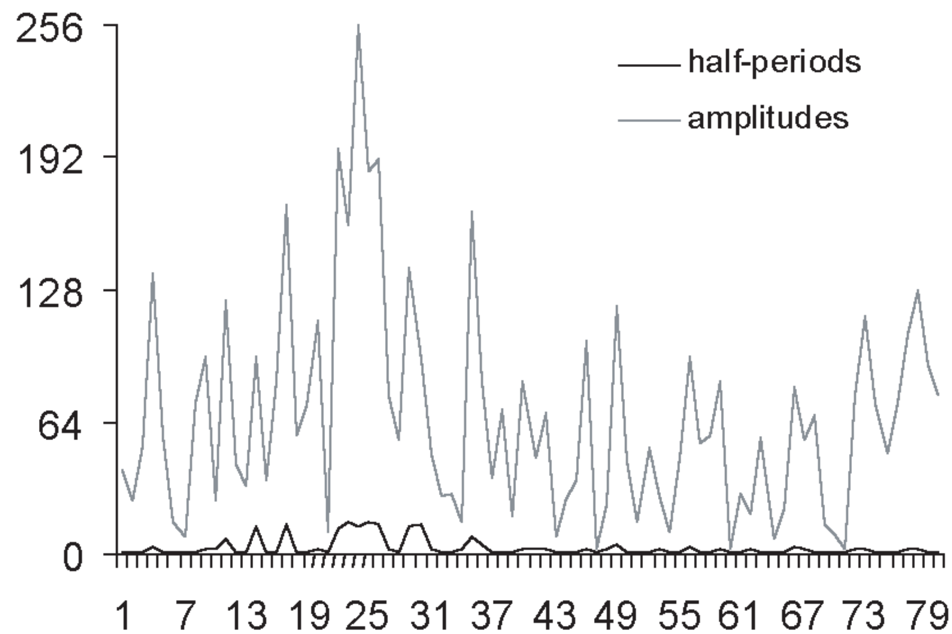

Figure 2: An example of filtration of a tsunami record.

3. Analytic Method of Data Analysis

3.1 Setting of the problem

One of the elements of technology of data processing from a deep ocean network of tsunami detection buoys is determination of the character of sea bed perturbation in the earthquake epicenter.

Mathematically, in the frames of the shallow water approximation, the problem can be formulated as follows (Avdeev and Goruynov, 1996; Avdeev *et al.*, 1999; Vasiliev, 1981):

$$\frac{\partial^2 u(x,y)}{\partial t^2} = div\,(D(x,y)\cdot\nabla u(x,y)) + \frac{\partial^2 f(t;x,y)}{\partial t^2}\ , \tag{8}$$

where $u(x,y)$ is the wave height, $D(x,y)$ is the known profile of depth, and $f(t;x,y)$ is the sea bed perturbation that generates a tsunami.

Assume the special structure of the source term, namely, that $f(t;x,y) = \theta(t)\cdot\varphi(x,y)$, where $\theta(t)$ is the Heaviside function.

It is required to reconstruct the sea bed perturbation function $\varphi(x,y)$, whose localization is approximately known, if we know "mareograms," i.e., readings of deep ocean sensors located at known points:

$$u(x_i,y_i,t) = \eta_i(t), \quad i = 1,\dots,n.$$

In such a statement we obtain one of the known inverse problems of mathematical physics: the source reconstruction problem. This problem in its general form can hardly be solved effectively, but one knows the representation of the source in the form of the product of spatial and temporal components, the latter of which are representable in terms of the delta-function.

3.2 Description of the numerical algorithm

For the case of a space distributed source, there are practically no stable numerical algorithms in the practice of solution of inverse problems. This fact is due to the high complexity and computational cost of the problem. Most theoretical results also concern isolated special cases.

In this work, a numerical approach based on the search for extrema of misfit functionals of special form is applied. The scheme of the method is as follows. Using the initial approximation for the source function $f_s(x,y)$ (constructed by *a priori* data, obtained from seismic observations), we solve the direct problem (8), i.e., we find the field of wave heights $u_s(x,y,t)$. Then we seek for the minimum point of the misfit functional, the latter being defined as the root-mean-square deviation of the measured data $\eta_i(t)$ and computed data $u_s(x_i,y_i,t)$. For this purpose we analytically construct the Green function of the problem, which is formally conjugate to the problem (8).

Preliminary numerical experiments have been carried out for 1-D (in the space variable) setting when there is no dependence on y, i.e., in the frames of processing of one trace. The source is approximated by the finite sum

$$\varphi(x) = \begin{cases} 0 & x < x_b \\ \sum_{k=1}^{N} A_k \sin(k\pi \frac{x-x_b}{x_e-x_b}), & x_b < x < x_e \\ 0 & x > x_e \end{cases} \tag{9}$$

where x_b and x_e are the coordinates of the domain of formation of the earthquake source.

3.3 Numerical experiments

For a numerical experiment, we have used the profile of the ocean bed near the shore of Chile in the region of the epicenter of the disastrous tsunami that happened on 5 May 1960 (see Fig. 3). The 1D ocean bottom profile is shown in Fig. 4, where the supposed epicenter of the tsunamigenic earthquake is marked by a dashed line and the location of the detection buoy is marked by a dot. The x-axis is directed toward the shore; the zero point lies at a distance of 136.5 km from the shore.

Three different types of the source have been considered:

(a) abrupt upthrust in the direction of the shore; in this case the source was taken in the form

$$\varphi(x) = -0.05 \cdot \sin\left(2\pi \frac{x - x_b^*}{x_e^* - x_b^*}\right) \cdot \left(2\pi \frac{x - x_b^*}{x_e^* - x_b^*}\right)^2, \tag{10}$$

where x_b^* and x_e^* delimit the actual region of formation of the earthquake center ($x_b \leq x_b^* < x_e^* \leq x_e$);

(b) abrupt upthrust in the direction of the sea; in this case the source was taken in the form

$$\varphi(x) = 0.05 \cdot \sin\left(2\pi \frac{x - x_b^*}{x_e^* - x_b^*}\right) \cdot \left(2\pi \frac{x_e^* - x}{x_e^* - x_b^*}\right)^2; \tag{11}$$

(c) elliptical uprising; in this case the source was taken in the form of the function

$$\varphi(x) = 0.8 \cdot \left(1 - \frac{\left(x - \left(x_e^* + \frac{x_e^*-x_b^*}{2}\right)\right)^2}{\left(\frac{x_e^*-x_b^*}{2}\right)^2}\right). \tag{12}$$

The effective size of the source $x_e^* - x_b^*$ was set equal to 14 km, and the size of the region of expected location of the epicenter $x_e - x_b$ was set equal to 25 km (i.e., twice as large).

Table 1 presents a number of parameters characterizing the field of permanent displacements created by each of the above sources, in particular, the values of the maximal positive and negative vertical displacements

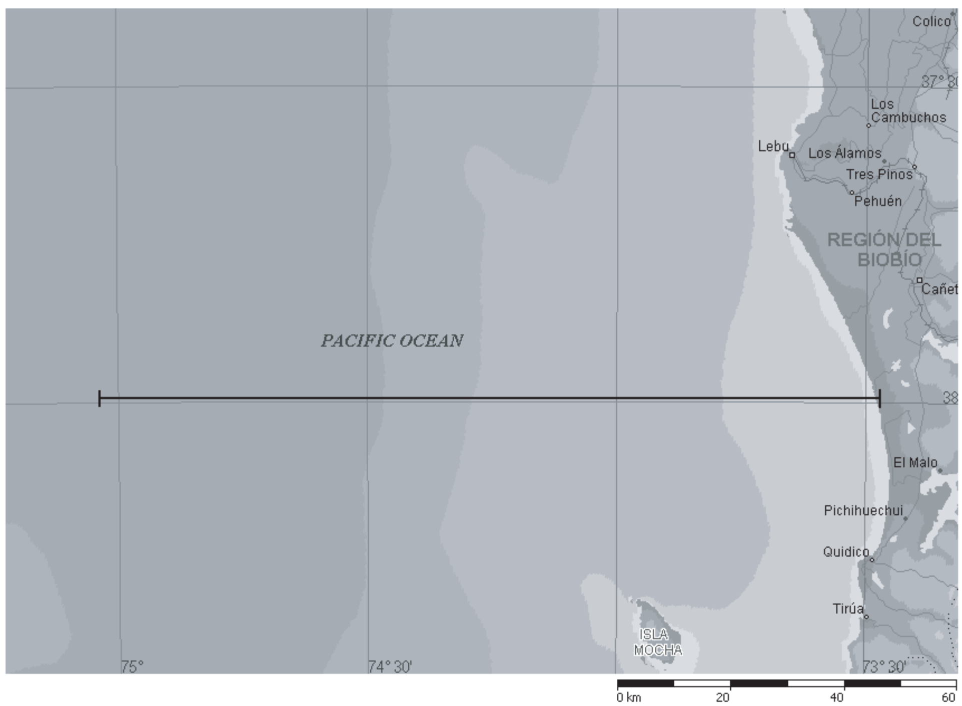

Figure 3: Region of the earthquake on 5 May 1960, approximates trace is marked by the solid line.

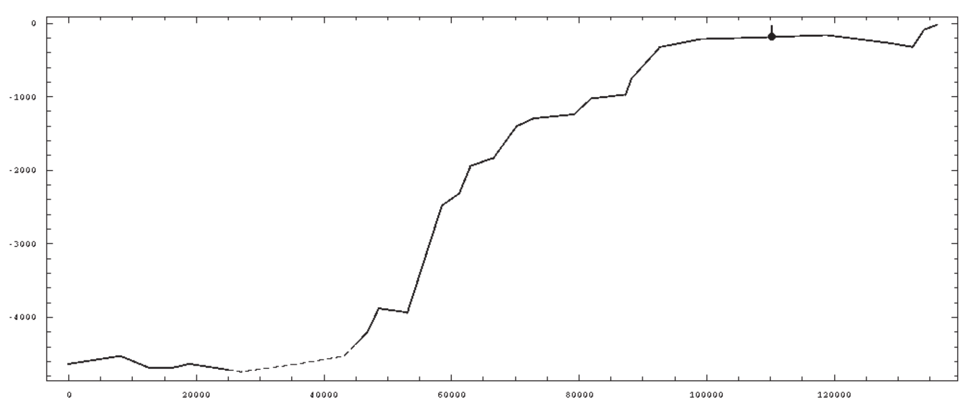

Figure 4: The trace profile. The supposed epicenter is marked by a dashed line and the location of the detection buoy is marked by a dot.

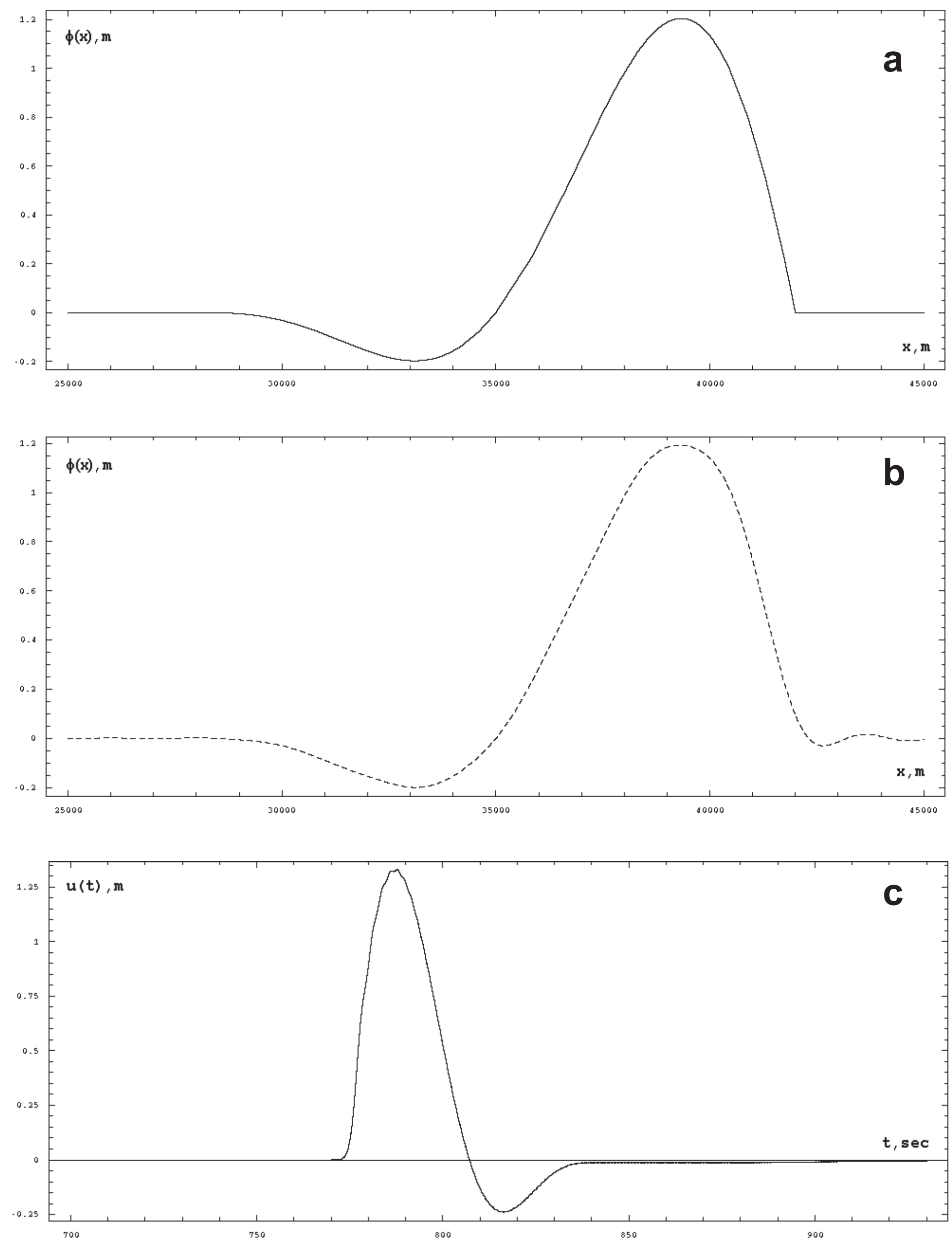

Figure 5: Abrupt upthrust in the direction of the shore from (a) the actual model source, (b) the reconstructed source, and (c) the form of the passing wave registered at the detection buoy.

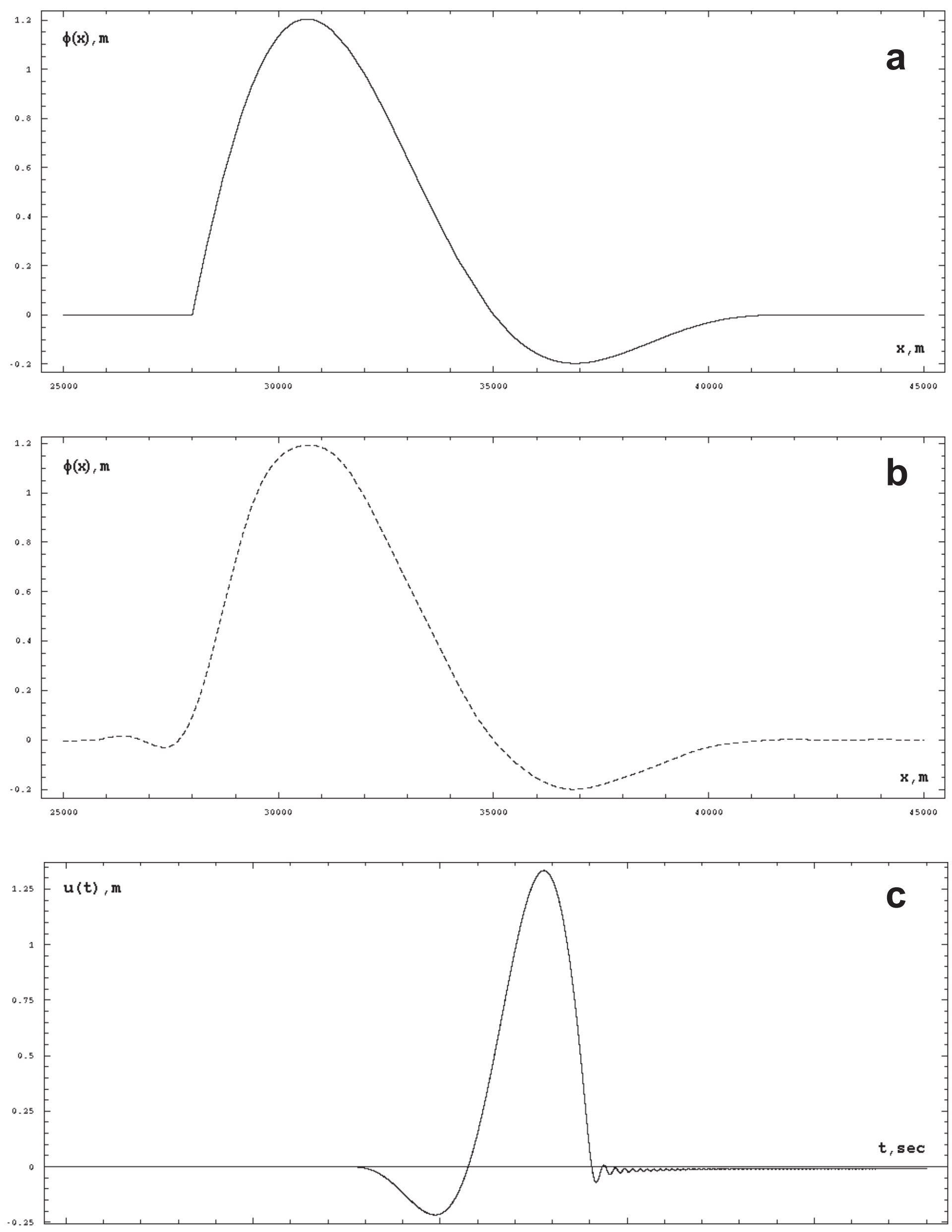

Figure 6: Abrupt upthrust in the direction of the sea from (a) the actual model source, (b) the reconstructed source, and (c) the form of the passing wave registered at the detection buoy.

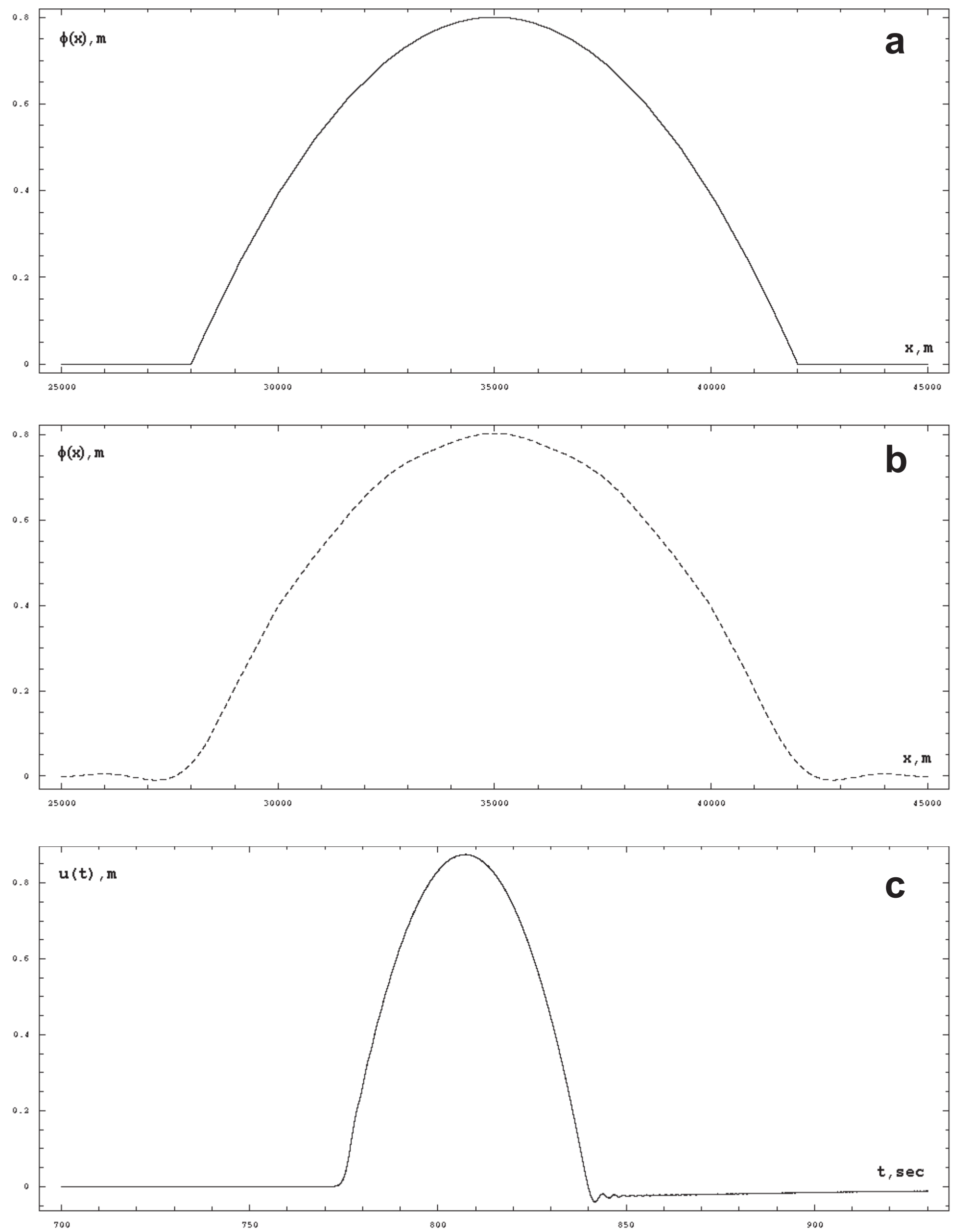

Figure 7: Elliptical uprising from (a) the actual model source, (b) the reconstructed source, and (c) the form of the passing wave registered at the detection buoy.

Table 1: Values characterizing the waves that are aroused by the source in the coastal area.

Source	$U_z{}^+$, m	$U_z{}^-$, m	ΔU_z, m	ΔV, m^2	V_0, m^2	E_t, MJ	$h_{\max}$, m	K_m
(a)	1.20	1.20	1.39	4398.2	3952.6	23.49	1.329	1.912
(b)	0.19	0.19	1.39	4398.2	3952.6	23.49	1.333	1.918
(c)	0.80	0.0	0.80	7466.6	7466.6	23.43	0.875	2.187

U_z^+ and U_z^-, the value of the maximal amplitude of the ocean bed displacement $\Delta U_z = U_z^+ + U_z^-$, and also the change of the basin's volume $\Delta V = \int_{x_b}^{x_e} U_z(x)dx$ and the total volume of displacement of the ocean bed $V_0 = \int_{x_b}^{x_e} |U_z(x)|dx$. The values of the tsunami's initial energy E_t are also presented. The latter values were calculated in statistical approximation by the formula $E_t = \frac{\rho_w g}{2} \int_{x_b}^{x_e} U_z^2(x)dx$, where ρ_w is the density of water. Since the one-dimensional case is considered, all the parameters are given per one linear meter of the wave front propagation.

Table 1 also contains values characterizing the waves that are aroused by the source in the coastal area, in particular, the values of the maximal heights of waves in the vicinity of the detection buoy $h_{\max}$ and the values of the wave amplification coefficient K_m, which is defined as the ratio of the maximal wave height in the area of detection buoy to the maximal wave height in the area of the source: $K_m = \frac{2 \cdot h_{\max}}{\Delta U_z}$.

4. Conclusion

A variant of the modern informational-computational technologies for analysis of tsunami observation data is proposed. These technologies are capable of effectively solving the posed problems in on-line mode in local systems of tsunami early detection and warning in the Pacific Ocean.

5. References

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